DIFFERENTIATION AND THEIR APPLICATION

Derivative or differential coefficient of a function: If $y = f(x)$ is a function of x and δx, a small change in the value of x, then $\underset{\delta x \to 0}{\text{Lt}} \frac{\delta y}{\delta x}$

$= \underset{\delta x \to 0}{\text{Lt}} \frac{f(x+\delta x) - f(x)}{\delta x}$ if it exists, is called the differential coefficient or derivative of $y = f(x)$ and is denoted by $\frac{dy}{dx}$ or $f'(x)$.

Note 1. $\frac{dy}{dx}$ is read as 'dee y by dee x'.

The symbol $\frac{d}{dx}$ stands for the operation of differentiation with respect to x. Thus $\frac{dy}{dx} = \frac{d}{dx}(y)$

Note 2. The process of finding $\frac{dy}{dx}$ is called differentiation

Note 3. The process of finding the derivative of a function by using the definition of derivative as a limit is called *differentiation from first principles* or *differentiation ab-initio or differentiation by delta method.*

Work Rule

1. Denote the given function by y i.e., let $y = f(x)$.
2. Let δx be a small change in x and δy then the corresponding change in y, so that $y + \delta y = f(x + \delta x)$.
3. Find δy by subtracting y from $y + \delta y$. Thus $\delta y = f(x + \delta x) - f(x)$.
4. Divide both sides by δx to obtain the difference quotient

   $$\frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}.$$

5. Find the limit of $\frac{\delta y}{\delta x}$ as $\delta x \to 0$

   $$\frac{dy}{dx} = \underset{\delta x \to 0}{\mathrm{Lt}} \frac{\delta y}{\delta x}$$

Derivative of x^n where $n \in R$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

[Write power before x and subtract one from the power.]

e.g. $\frac{d}{dx}\left(x^8\right) = 8x^{8-1} = 8x^7$

Note. $\frac{d}{dx}(x) = 1$ *i.e.* rate of change of any variable w.r.t itself is 1.

Caution. $\frac{d}{dx}(y)$ is $\frac{dy}{dx}$ and not 1.

Fundamental theorems on differentiation:

Theorem 1. Derivative of a constant is zero.

i.e. $\frac{d}{dx}(c) = 0$

e.g., $\frac{d}{dx}(\pi) = 0; \frac{d}{dx}(9) = 0.$

Theorem 2. The derivative of the product of a constant and a function is equal to the product of the constant and the derivative of the function.

i.e. $$\frac{d}{dx}[c f(x)] = c\frac{d}{dx}[f(x)]$$

e.g., $$\frac{d}{dx}(5x^7) = 5\frac{d}{dx}(x^7) = 5(7x^{7-1})$$
$$= 5(7x^6) = 35x^6$$

Theorem 3. The derivative of the algebraic sum of any finite number of functions is the algebraic sum of their derivatives.

i.e. $$\frac{d}{dx}[f(x) + g(x) - h(x) + \ldots]$$

$$= \frac{d}{dx}f(x) + \frac{d}{dx}g(x) - \frac{d}{dx}h(x) + \ldots$$

e.g., $$\frac{d}{dx}(x^3 - 4x + 8)$$

$$= \frac{d}{dx}(x^3) - \frac{d}{dx}(4x) + \frac{d}{dx}(8)$$

$$= 3x^2 - 4 + 0 = 3x^2 - 4.$$

Theorem 4. An additive constant disappears in differentiation.

i.e. $$\frac{d}{dx}[f(x)+c]=\frac{d}{dx}[f(x)].$$

where c is a constant.

e.g., $$\frac{d}{dx}[x^{10}+16]=\frac{d}{dx}x^{10}=10x^{9}$$

Theorem 5. Derivative of product of two functions; the differential coefficient of the product of two functions is the sum of the products of each function with the derivative of the other.

i.e., $$\frac{d}{dx}(u.v)=u\frac{dv}{dx}+v\frac{du}{dx}$$

where u and v are differential functions of x.

Working Rule: The differential coefficient of the product of two functions = first function × the differential coefficient of the second + second function × the differential coefficient of the first.

Cor. $$\frac{d}{dx}(uvw)=uv\frac{dw}{dx}+uw\frac{dv}{dx}+vw\frac{du}{dx}$$

where u, v, w are functions of x and their derivative exists.

Theorem 6. The differential coefficient of the quotient of two functions is

$$i.e., \quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

where u and v are derivable functins of x

In words

$$= \frac{\text{Denom} \times \text{Derivative of Num} - \text{Num} \times \text{Derivative of Denom}}{(\text{Denom})^2}$$

Derivatives of Trigonometric Functions:

$\frac{d}{dx}\sin x = \cos x$ $\quad\quad$ $\frac{d}{dx}\sin u = \cos u \times \frac{du}{dx}$

$\frac{d}{dx}\cos x = -\sin x$ $\quad\quad$ $\frac{d}{dx}\cos u = -\sin u \times \frac{du}{dx}$

$\frac{d}{dx}\tan x = \sec^2 x$ $\quad\quad$ $\frac{d}{dx}\tan u = \sec^2 u \times \frac{du}{dx}$

$\frac{d}{dx}\cot x = -\text{cosec}^2 x$ $\quad\quad$ $\frac{d}{dx}\cot u = -\text{cosec}^2 u \times \frac{du}{dx}$

$\frac{d}{dx}\sec x = \sec x \tan x$ $\quad\quad$ $\frac{d}{dx}\sec u = \sec u \tan u \times \frac{du}{dx}$

$\frac{d}{dx}\text{cosec}\, x = -\text{cosec}\, x \cot x$ $\quad\quad$ $\frac{d}{dx}\text{cosec}\, u = -\text{cosec}\, u \cot u \times \frac{du}{dx}$

Note:

(*i*) Differential coefficients of those trigonometrical ratios which begins with "co" are negative.

(*ii*) Never forget to multiply by the differential coefficient of u *i.e.*, angle *e.g.*, $\frac{d}{dx}[\sin 6x]$ $= \cos 6x \times 6 = 6\cos 6x \cdot$

Properties of Inverse Trigonometric Functions:

1. *Some angle can be expressed by different inverse trigonometric functions.* We know that

$$\sin 60° = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad 60° = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\cos 60° = \frac{1}{2} \quad \Rightarrow \quad 60° = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\tan 60° = \sqrt{3} \quad \Rightarrow \quad 60° = \tan^{-1}\left(\sqrt{3}\right)$$

and so on, $60° = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \cos^{-1}\left(\frac{1}{2}\right)$ $= \tan^{-1}\left(\sqrt{3}\right) = \ldots..$

2. Inverse property. We know that if $x = \cos\theta$ then, $\theta = \cos^{-1} x$.

$\therefore\ \theta = \cos^{-1}(\cos\theta)$ $\qquad (\because\ x = \cos\theta)$

3. Principle of reciprocity

(*i*) $\text{cosec}^{-1}\dfrac{1}{x} = \sin^{-1} x$

(*ii*) $\sec^{-1}\dfrac{1}{x} = \cos^{-1} x$

(*iii*) $\cot^{-1}\dfrac{1}{x} = \tan^{-1} x$

4. *Inverse trigonometric functions are odd functions within the principle value i.e.,*

(*i*) $\sin^{-1}(-x) = -\sin^{-1} x$

(*ii*) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$

(*iii*) $\tan^{-1}(-x) = -\tan^{-1} x$

5. *Some fundamental Formulae*

(*i*) $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$

(*ii*) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

(*iii*) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$

(*iv*) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(*v*) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(*vi*) $2\tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} = \cos^{-1} \frac{1-x^2}{1+x^2}$

$$= \tan^{-1} \frac{2x}{1-x^2}$$

6. To express one inverse trigonometric function in terms of another one *i.e.*,

(*i*) $\sin^{-1} x = \cos^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$

(*ii*) $\cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{\sqrt{1-x^2}}{x}$

(*iii*) $\operatorname{cosec}^{-1} \frac{1}{x} = \sec^{-1} \frac{1}{\sqrt{1-x^2}} = \cot^{-1} \frac{\sqrt{1-x^2}}{x}$

Derivatives of inverse trigonometric functions:

(i) $$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx}(u)$$

(ii) $$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} u\right) = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{d}{dx}(u)$$

(iii) $$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2} \cdot \frac{d}{dx}(u)$$

(iv) $$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\cot^{-1} u\right) = \frac{-1}{1+u^2} \cdot \frac{d}{dx}(u)$$

(*v*) $$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{1}{u\sqrt{1-u^2}} \cdot \frac{d}{dx}(u)$$

(*vi*) $$\frac{d}{dx}\left(\operatorname{cosec}^{-1} x\right) = \frac{-1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}\left(\operatorname{cosec}^{-1} u\right) = \frac{-1}{u\sqrt{1-u^2}} \cdot \frac{d}{dx}(u)$$

Some properties of logarithms:

(*i*) $\log_a mn = \log_a m + \log_a n$

(*ii*) $\log_a \frac{m}{n} = \log_a m - \log_a n$

(*iii*) $\log_a m^n = n \log_a m$

(*iv*) $\log_a m \log_b a = \log_b m$

(*v*) $\log_a b . \log_b a = 1.$

Where m, n, a and b are positive real numbers.
Logarithmic Differentiation: If u is a function of x, then

$$\frac{d}{dx}\left(u^n\right) = nu^{n-1}\frac{du}{dx}, \frac{d}{dx}\left(a^u\right) = a^u \log a \frac{du}{dx}$$

In this case, the power is a constant and in the second case, the base is constant.

To differentiate u^v where u and v both are functions of x, we first take logarithms of both sides and then differentiate. This process is called logarithmic differentiation. This process is also useful when the function consists of the product or quotient of a number of functions.

Example. Differentiate $(1+x)^{2x}\, w.r.t.x$

Sol. Let $y = (1+x)^{2x}$ [Form u^v]

Taking log to both sides

$$\log y = \log\,(1+x)^{2x} = 2x\log(1+x)$$

$$\left[\left(\log m^n = n\log m\right)\right]$$

Differentiating w.r.t.x, we get

$$\frac{1}{y}\frac{dy}{dx} = 2x\frac{d}{dx}\left[\log(1+x)\right] + \log(1+x).\frac{d}{dx}(2x)$$

$$= 2x.\frac{1}{x+1} + \log(1+x)(2)$$

$$\frac{dy}{dx} = y\left[\frac{2x}{x+1} + 2\log(1+x)\right]$$

Putting the value of y

$$\frac{dy}{dx} = 2(1+x)^{2x}\left[\frac{x}{1+x} + \log(1+x)\right]$$

Function of a function : If y a function of u and u in turn is a function of x then y is called a *function of a function, or a composite function.*

Theorem : If y is a function of u and u in turn is a function of x, then y is a function of x and

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} \text{ (Chain Rule)}$$

Note 1. *Extension of Chain Rule*

If $y = f(u)$, $u = g(u)$ and $v = h(x)$

then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$

Note 2. If $y = u^n$ and $u = g(x) =$ a function of x.

$$\Rightarrow \frac{dy}{dx} = nu^{n-1}\frac{d}{dx}(u)$$

Aid to memory. $\frac{d}{dx}$ [A function of $x]^n$ = $n(\text{function})^{n-1} \times$ diff. coeff. of the function $w.r.t.x$.

Theorem. $\frac{dy}{dx} \times \frac{dx}{dy} = 1.$

Diff. parametric equations. If x and y be expressed in terms of any variable parameter t then

$$\frac{dy}{dx} = \frac{dy / dt}{dx / dt}$$

Second Derivative : If $y = f(x)$ is a differentiable function of x, then its derivative $\frac{dy}{dx}$ is also a function of x. If the function $\frac{dy}{dx}$ of x is also differentiable, then its derivative is denoted by $\frac{d^2y}{dx^2}$ or $f''(x)$ or $\frac{d^2y}{dx^2}$, is called the seconed derivative of $y = f(x)$. $\frac{d^2y}{dx^2}$ is read as dee two y over dee x squared. $\frac{d^2y}{dx^2}$ is also denoted by y'' or y_2.

Illustration : If $y = \sin^{-1}x$, prove that

$$\frac{d^2y}{dx^2} = \frac{x}{\left(1-x^2\right)^{3/2}}$$

Sol. We have $y = \sin^{-1} x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} = \left(1-x^2\right)^{-1/2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\left(1-x^2\right)^{-1/2}\right]$$

$$= -\frac{1}{2}\left(1-x^2\right)^{-3/2}\frac{d}{dx}\left(1-x^2\right)$$

$$= -\frac{1}{2}\frac{1}{\left(1-x^2\right)^{3/2}}(-2x) = \frac{x}{\left(1-x^2\right)^{3/2}}$$

The nth Derivative : If $y = f(x)$ is a differentiable function of x, then its nth derivative may or may not exists. For $n(>1) \in \mathrm{N}$, the nth derivative of y exists if the $(n-1)$th derivative of y is differentiable. For example for $n = 3$, the 3rd derivative of $y = f(x)$ exists if $\frac{d^2y}{dx^2}$ is differentiable.

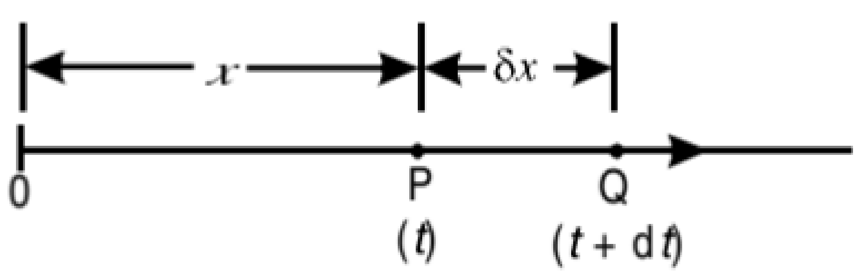
If the 3rd derivative of $y = f(x)$ is differentiable, then we can talk of its fourth derivative. The nth derivative of $y = f(x)$ is denoted by

$$\frac{d^n y}{dx^n} \text{ or } f^{(n)}(x), y_n, D^n y \text{ etc.}$$

Application of Differentiation

Motion in a Straight line: Let O be a fixed point on a straight line OX. Let P be the position at time t and Q, the position of the particle at time $t + \delta t$. Let OP = x and OQ = $x + \delta x$. Therefore displacement of the particle in time $(t + \delta t) - t = \delta t$ is given by $PQ = (x + \delta x) - x = \delta x$. The ratio $\frac{\delta x}{\delta t}$ is called the average velocity of the particle between P and Q.

The limit of average velocity $\frac{\delta x}{\delta t}$ as $\delta t \to 0$ is defined as the velocity of the particle P.



Velocity of particle at P = $\lim_{\delta t \to 0} \frac{\delta x}{\delta t} = \frac{dx}{dt}$

The velocity is denoted by $v = \frac{dx}{dt}$

Note 1. If $v = \frac{dx}{dt} > 0$, then the particle moves in the direction of x increasing, because in the case δx and δt will have same sign.

Note 2. If $v = \frac{dx}{dt} < 0$, then the particle moves in the direction of x decreasing, because δx and δt will have opposite signs.

Note 3. If $v = \frac{dx}{dt} = 0$, then the particle is instantaneously at rest.

Acceleration : The ratio $\frac{\delta v}{\delta t}$ is called the average rate of change of velocity (or average acceleration) of the particle between the points P and Q.

Acceleration of particle at P = $\lim_{\delta t \to o} \frac{\delta v}{\delta t} = \frac{dv}{dt}$

$$\therefore a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Note 1. If $a = \frac{dv}{dt} > 0$, then the velocity of particle is increasing.

Note 2. If $a = \frac{dv}{dt} < 0$, then the velocity of particle is decreasing.

Motion Under Gravity: The acceleration of the falling body due to gravity towards the centre of earth is denoted by g. Its value is $g = 32$ ft/sec^2 or 9.8 metres/sec^2. For upward motion g is taken as –ve and for downward motion it is taken as +ve.

Rate of change of quantities: Let x and y be any variables and y, a function of x. Therefore, an increased δx in the value of x shall cause an increment δy (say) in the value of y.

We have seen that the ratio $\frac{\delta y}{\delta t}$ is called the average rate of change of y *w.r.t.* x and limit of $\delta y/\delta x$ as $\delta x \to 0$ is the instantaneous rate of change of y *w.r.t.x*.

If x and y are both functions of parameter t, then

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$\therefore$ If the rate of change of x w.r.t. t is known, then we can find the value of rate of change of y w.r.t. t

Increasing and Decreasing functions: Let I be an open interval contained in the domain of a real function f.

(*i*) $f(x)$ is called an increasing function on I.

if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$

(*ii*) $f(x)$ is called a decreasing function on I.

if $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$

Again

(*iii*) A function f is said to be increasing at a point x_0, if there is an interval $I = (x_0 - h, x_0 + h)$ around x_0 such that for $x_1, x_2 \in$ I

$$x_0 < x_2 \Rightarrow f(x_0) < f(x_2)$$

and $$x_1 < x_2 \Rightarrow f(x_1) < f(x_0)$$

(*iv*) A function f is said to be decreasing at a point x_0, if there is an interval I = $(x_0 - h, x_0 + h)$ around x_0 such that x_1, x_2 I

$$x_0 < x_0 \Rightarrow f(x_0) > f(x_2)$$

and $$x_1 < x_0 \Rightarrow f(x_1) > f(x_0)$$

Note 1. The same function can be increasing function in a certain interval and decreasing function in certain other interval.

Note 2. Certain functions are neither increasing nor decreasing in a given interval.

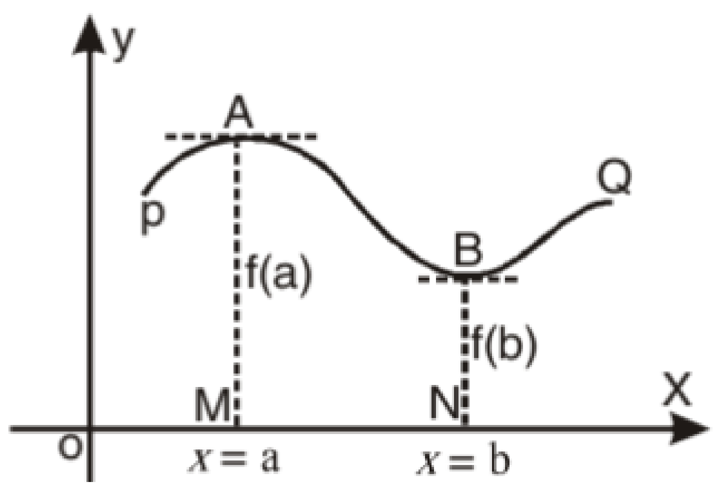
Theorem 1. A differentiable real function $f(x)$ is increasing on an open interval I if and only if $f'(x) > 0$ for all x in I.

Theorem 2. A differentiable real function is decreasing on an interval if $f'(x)$ for all x in I.

Maxima and Minima

1. A function $f(x)$ is said to be maximum at $x = a$ if $f(a)$ is the greatest value of $f(x)$ in the immediate neighbourhood of $x = a$.

   Graphically: $f(x)$ is maximum at A and its maximum value is AM = $f(a)$



2. A function $f(x)$ is said to be minimum at $x = a$ if (a) is the least value of $f(x)$ in the immediate neighbourhood of $x = a$.

Graphically : $f(x)$ is minimum at B and its minimum value is $BN = f(b)$.

3. Maximum and minimum values of a function are called extreme values. The points A and B are called stationary points or turning points or points of extreme values.

Necessary and sufficient condition for maximum and minimum values.

Working Rule. For finding the maximum and minimum value of $y = f(x)$.

(*i*) Put $\frac{dy}{dx} = 0$. Solve it for x, giving $x = a, b, c, ...$

(*ii*) Select $x = a$, study the sign of $\frac{dy}{dx}$ when

(*i*) $x < a$ slightly

(*ii*) $x > a$ slightly

(*a*) If the former is +ve and later is –ve then $f(x)$ is maximum at $x = a$.

(*b*) If the former is –ve and later is +ve, then $f(x)$ is min., at $x = a$.

(*iii*) Putting these values of x for which $f(x)$ is max. or min. and get the corresponding max. or min. values of $f(x)$.

Use of second derivative Theorem

1. A function $f(x)$ is maximum at $x = a$ if $f'(a) = 0$ and $f''(a) < 0$.
2. A function $f(x)$ is minimum at $x = a$ if $f'(a) = 0$ and $f''(a) > 0$. (+ve)

Working rule to find the max. or min. values

(*i*) Put y = given function $f(x)$ and find $\frac{dy}{dx}$ *i.e.*, $f'(x)$.

(*ii*) Put $\frac{dy}{dx} = 0$ *i.e.*, $f'(x) = 0$ and solve it for x giving $x = a, b, c...$

(*iii*) Select $x = a$, find $\frac{d^2y}{dx^2}$ *i.e.*, $f''(x)$ at $x = a$

(*a*) If $\left(\frac{d^2y}{dx^2}\right)_{x=a}$ i.e., $f''(a)$ is –ve, $x = a$ gives the max. values of the function.

(*b*) If $\left(\frac{d^2y}{dx^2}\right)_{x=a}$ i.e., $f''(a)$ is +ve, $x = a$ gives the min. values of the function.

Rolle's theorem: If a function $f(x)$ is

(*i*) Continuous in the closed interval $[a, b]$ *i.e.* $a \leq x \leq b$

(*ii*) derivable in the open interval (a, b) *i.e.*, $a < x < b$

(*iii*) $f(a) = f(b)$

then there exists at least one point c in the open interval (a, b) (i.e., $a < c < b$) such that $f'(c) = 0$.

Aid to memory

(*i*) Rolle's theorem fails for the function which does not even satisfies one condition.

(*ii*) Every polynomial in x is a continuous function for each x.

$\sin x$, $\cos x$, e^x are continuous for all values of x.

$\log x$ is continuous for all $x > 0$.

(*iii*) If f and g are both continuous on the closed interval $[a, b]$ then $f \pm g$ and fg are also continuous on $[a, b]$

(iv) If $f(x)$ is derivable for every point in a given interval, then it must be continuous in this interval.

i.e., **Derivability** $\Rightarrow$ continuity.

Lagrange's mean value theorem: If a function $f(x)$ is, (*i*) continuous in the closed interval $[a, b]$ *i.e.*, $a \le x \le b$

(*ii*) derivable in the open interval (a, b) *i.e.*, $a < x < b$

then, there exists at least one point c in the open interval (a, b) $[a < c < b]$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c),$$

Graph of functions: The graph of function $y = f(x)$,

1. Find whether the curve is increasing or decreasing.
   Also, find the turning points, if any
2. Symmetry
   (*i*) Find whether the curve is symmetrical about the x-axis.
   This will happen if no change is affected if y is changed to $-y$. *e.g.*, $y^2 = 4\,ax$ is symmetrical about x-axis.
   [$\because$ only even powers of y occurs]
   (*ii*) Find whether the curve is symmetrical about the y-axis.

This will happen if no change is affected if x is changed to $-x$ *e.g.*, $x^2 = 4\ ay$ is symmetrical about y-axis.

[$\because$ only even powers of x occurs]

Note : $x^2 + y^2 = a^2$ is symmetrical about both axis.

(*iii*) Find whether the curve is symmetrical in opposite quadrants. This will happen if no change is affected if x is changed to $-x$ and y to $-y$ *e.g.*, $xy = k$ is symmetrical in opposite quadrants.

(*iv*) Table. Form a table by taking suitable values of x and y.

(*v*) Plot the above points and join them by free hand drawing so as to get the required rough sketch.

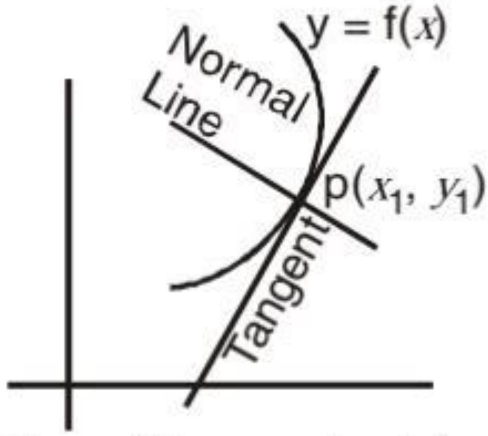
Tangents and Normals: Equations of the tangent and the normal to the curve.

Here, the equation of the curve is

$$y = f(x)$$

$$\frac{dy}{dx} = \text{slope of tangent at } (x, y)$$

$$\therefore \text{ Slope of tangent at } P(x_1, y_1) = \left(\frac{dy}{dx}\right)_{\text{at}(x_1,\, x_2)} = m$$



The equation of tangent at (x_1, y_1) is

$y - y_1 = m(x - x_1)$

Slope of normal line at a given point is the negative of the reciprocal of the slope of the tangent line at that point.

Slope of normal at $P(x_1, y_1) = -\frac{1}{m}(m \neq 0)$.

The equation of the normal at (x_1, y_1) is

$(y - y_1) = -\frac{1}{m}(x - x_1)$.